Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **May/June 2021**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **16** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	State or imply non-modular inequality $2^2(3x-1)^2 < (x+1)^2$, or corresponding quadratic equation, or pair of linear equations	**B1**	
	Form and solve a 3-term quadratic, or solve two linear equations for x	**M1**	e.g. $35x^2 - 26x + 3 = 0$
	Obtain critical values $x = \frac{3}{5}$ and $x = \frac{1}{7}$	**A1**	Allow 0.143 or better
	State final answer $\frac{1}{7} < x < \frac{3}{5}$	**A1**	Exact values required. Accept $x > \frac{1}{7}$ **and** $x < \frac{3}{5}$ Do not condone ⩽ for < in the final answer. Fractions need not be in lowest terms.
	Alternative method for Question 1		
	Obtain critical value $x = \frac{3}{5}$ from a graphical method, or by solving a linear equation or linear inequality	**B1**	
	Obtain critical value $x = \frac{1}{7}$ similarly	**B2**	Allow 0.143 or better
	State final answer $\frac{1}{7} < x < \frac{3}{5}$	**B1**	OE. Exact values required. Accept $x > \frac{1}{7}$ **and** $x < \frac{3}{5}$ Do not condone ⩽ for < in the final answer. Fractions need not be in lowest terms.
		4	

Question	Answer	Marks	Guidance
2	Reduce to a 3-term quadratic $u^2+6u-1=0$ OE	**B1**	Allow '= 0' implied
	Solve a 3-term quadratic for u	**M1**	
	Obtain root $\sqrt{10}-3$	**A1**	
	Obtain answer $x = -1.818$ only	**A1**	The question asks for 3 d.p.
	Reject $-\sqrt{10}-3$ correctly	**B1**	e.g. by stating that $e^x>0$ or $\ln\left(-10-\sqrt{3}\right)$ is impossible Not "math error".
	Alternative method for Question 2		
	Rearrange to obtain a correct iterative formula	**B1**	e.g. $x_{n+1}=-\ln\left(6+e^{x_n}\right)$
	Use the iterative process at least twice	**M1**	
	Obtain answer $x = -1.818$	**A1**	
	Show sufficient iterations to at least 4 d.p. to justify $x = -1.818$	**A1**	$1, -2.165..., -1.811..., -1.819..., -1.818..., -1.818...$
	Clear explanation of why there is only one real root	**B1**	
		5	

Question	Answer	Marks	Guidance
3(a)	Use correct trig expansions and obtain an equation in $\sin x$ and $\cos x$	*M1	
	Use correct exact trig ratios for $30°$ in *their* expansion	B1 FT	e.g. $\cos x\left(\frac{\sqrt{3}}{2}-1\right)=\sin x\left(\sqrt{3}-\frac{1}{2}\right)$
	Obtain an equation in $\tan x$	DM1	Allow if their error in line 1 was a sign error
	Obtain $\tan x=\dfrac{2-\sqrt{3}}{1-2\sqrt{3}}$ from correct working	A1	AG
		4	
3(b)	Obtain answer in the given interval, e.g.173.8°	B1	Accept $174°,\ 354°$ or better
	Obtain a second answer and no other in the given interval, e.g. 353.8°	B1	Ignore answers outside the given interval. Treat answers in radians (3.03 and 6.17) as a misread.
		2	

Question	Answer	Marks	Guidance
4(a)	Use correct double angle formula or t-substitution twice	M1	
	Obtain $\dfrac{1-\cos 2\theta}{1+\cos 2\theta}=\tan^2\theta$ from correct working	A1	AG
		2	

Question	Answer	Marks	Guidance
4(b)	Express $\tan^2\theta$ in terms of $\sec^2\theta$	M1	$\left(\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\left(\sec^2\theta \pm 1\right)\mathrm{d}\theta\right)$
	Integrate and obtain terms $\tan\theta - \theta$	A1	Accept with a mixture of x and θ
	Substitute limits correctly in an integral of the form $a\tan\theta + b\theta$, where $ab \neq 0$	M1	$\left(\sqrt{3} - \frac{\pi}{3} - \frac{1}{\sqrt{3}} + \frac{\pi}{6}\right)$ Allow if trig. not substituted
	Obtain answer $\frac{2}{3}\sqrt{3} - \frac{1}{6}\pi$	A1	or equivalent exact 2-term expression
		4	

Question	Answer	Marks	Guidance
5(a)	Use quadratic formula and $\mathrm{i}^2 = -1$	M1	
	Obtain answers $p\mathrm{i} + \sqrt{q - p^2}$ and $p\mathrm{i} - \sqrt{q - p^2}$	A1	Accept $\frac{2p\mathrm{i} \pm \sqrt{-4p^2 + 4q}}{2}$ and ISW
		2	
5(b)	State or imply that the discriminant must be negative	M1	
	State condition $q < p^2$	A1	
		2	

Question	Answer	Marks	Guidance
5(c)	Carry out a correct method for finding a relation, e.g. use the fact that the argument of one of the roots is $(\pm)60°$	M1	
	State a correct relation in any form, e.g. $\frac{p}{\sqrt{q-p^2}} = (\pm)\sqrt{3}$	A1	
	Simplify to $q = \frac{4}{3}p^2$	A1	
	Alternative method for Question 5(c)		
	Carry out a correct method for finding a relation, e.g. use the fact that the sides have equal length	M1	
	State a correct relation in any form, e.g. $4\left(q-p^2\right) = p^2 + q - p^2$	A1	
	Simplify to $q = \frac{4}{3}p^2$	A1	
		3	

Question	Answer	Marks	Guidance
6(a)	Use correct chain rule **or** correct quotient rule to differentiate x or y	**M1**	
	Obtain $\frac{dx}{dt} = \frac{3}{2+3t}$ **or** $\frac{dy}{dt} = \frac{2}{(2+3t)^2}$	**A1**	OE
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	**M1**	
	Obtain answer $\frac{2}{3(2+3t)}$	**A1**	OE. Express as a simple fraction but not necessarily fully cancelled.
	Explain why this is always positive	**A1**	For correct gradient. e.g. x is only defined for $2+3t>0$ hence gradient > 0
	Alternative method for Question 6(a)		
	Form equation in x and y only	**M1**	
	Obtain $y = \frac{e^x - 2}{3e^x}\left(= \frac{1}{3} - \frac{2}{3}e^{-x}\right)$	**A1**	OE
	Differentiate	**M1**	
	Obtain $y' = \frac{2}{3}e^{-x}$	**A1**	OE
	Explain why this is always positive	**A1**	
		5	

Question	Answer	Marks	Guidance
6(b)	Obtain $y = -\frac{1}{3}$ when $x = 0$	**B1**	
	Use a correct method to form the given tangent	**M1**	$\left(\frac{y+\frac{1}{3}}{x} = \frac{2}{3}\right)$
	Obtain answer $3y = 2x - 1$	**A1**	OE
		3	

Question	Answer	Marks	Guidance
7(a)	Use correct quotient rule or correct product rule	**M1**	e.g. $\frac{dy}{dx} = \frac{\sqrt{x}.\frac{1}{1+x^2} - \tan^{-1} x.\frac{1}{2\sqrt{x}}}{x}$
	Obtain correct derivative in any form	**A1**	
	Equate derivative to zero and remove inverse tangent	**M1**	
	Obtain $a = \tan\left(\frac{2a}{1+a^2}\right)$ from correct working	**A1**	AG. Accept with *x* in place of *a*.
		4	

Question	Answer	Marks	Guidance
7(b)	Calculate the value of a relevant expression or pair of expressions at $a = 1.3$ and $a = 1.5$	M1	Must be using radians
	Complete the argument correctly with correct calculated values	A1	e.g. $1.3 < 1.448, \quad 1.5 > 1.322 \quad (0.148, -0.178)$
		2	
7(c)	Use the iterative process $a_{n+1} = \tan\left(\frac{2a_n}{1+a_n{}^2}\right)$ correctly at least twice	M1	
	Obtain final answer 1.39	A1	
	Show sufficient iterations to at least 4 d.p. to justify 1.39 to 2 d.p. or show there is a sign change in the interval (1.385, 1.395)	A1	Allow recovery
		3	

Question	Answer	Marks	Guidance
8(a)	State or imply $\overrightarrow{AB} = \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix}$	B1	OE. Allow $\pm$
	Use the correct process to calculate the scalar product of a pair of relevant vectors, e.g. their $\overrightarrow{AB}$ and a direction vector for l	M1	$(2+2-3=1)$
	Using the correct process for the moduli, divide the scalar product by the product of the moduli of the two vectors and evaluate the inverse cosine of the result	M1	$\cos^{-1}\left(\frac{1}{\sqrt{6}\sqrt{14}}\right)$
	Obtain answer $83.7°$ or 1.46 radians	A1	Or answers rounding to $83.7°$ or 1.46 radians
		4	

Question	Answer	Marks	Guidance
8(b)	State or imply $\pm\ \overrightarrow{AP}$ and $\pm\ \overrightarrow{BP}$ in component form, i.e. $(1+\lambda, 1-2\lambda, \lambda)$ and $(-1+\lambda, 2-2\lambda, 3+\lambda)$, or equivalent	**B1**	
	Form an equation in λ by equating moduli or by using $\cos BAP = \cos ABP$	***M1**	
	Obtain a correct equation in any form $(1+\lambda)^2+(1-2\lambda)^2+\lambda^2=(\lambda-1)^2+(2-2\lambda)^2+(\lambda+3)^2$	**A1**	Or $(1+\lambda)\sqrt{14-4\lambda+6\lambda^2}=(13-\lambda)\sqrt{2-2\lambda+6\lambda^2}$ $\left(83\lambda^3-528\lambda^2+207\lambda-162=0\right)$
	Solve for λ and obtain position vector	**DM1**	[$\lambda = 6$]
	Obtain correct position vector for P in any form, e.g. (8, – 9, 7) or 8**i** – 9**j** + 7**k**	**A1**	Accept coordinates
		5	

Question	Answer	Marks	Guidance
9(a)	Use correct product rule or correct quotient rule	**M1**	
	Obtain correct derivative in any form	**A1**	$y'=\dfrac{x^{-\frac{2}{3}}}{x}-\dfrac{2}{3}x^{-\frac{5}{3}}\ln x$
	Equate 2 term derivative to zero and solve for x	**M1**	
	Obtain answer $x=e^{\frac{3}{2}}$	**A1**	Or exact equivalent
	Obtain answer $y=\dfrac{3}{2e}$	**A1**	Or exact equivalent
		5	

Question	Answer	Marks	Guidance
9(b)	Commence integration and reach $ax^{\frac{1}{3}}\ln x + b\int x^{\frac{1}{3}}.\frac{1}{x}\mathrm{d}x$	*M1	
	Obtain $3x^{\frac{1}{3}}\ln x - 3\int x^{\frac{1}{3}}.\frac{1}{x}\mathrm{d}x$	A1	
	Complete the integration and obtain $3x^{\frac{1}{3}}\ln x - 9x^{\frac{1}{3}}$	A1	OE
	Use limits correctly in an expression of the form $px^{\frac{1}{3}}\ln x + qx^{\frac{1}{3}}$ $(pq \neq 0)$	DM1	$6\ln 8 - 9\times 2 - 0 + 9$
	Obtain $18\ln 2 - 9$ from full and correct working	A1	AG need to see $\ln 8 = 3\ln 2$
		5	

Question	Answer	Marks	Guidance
10	State a suitable form of partial fractions for $\frac{1}{x^2(1+2x)}$	B1	e.g. $\frac{A}{x}+\frac{B}{x^2}+\frac{C}{1+2x}$ or $\frac{Ax+B}{x^2}+\frac{C}{1+2x}$
	Use a relevant method to determine a constant	M1	
	Obtain one of $A=-2$, $B=1$ and $C=4$	A1	
	Obtain a second value	A1	
	Obtain the third value	A1	
	Separate variables correctly and integrate at least one term	M1	
	Obtain terms $-2\ln x-\frac{1}{x}+2\ln(1+2x)$ and t	B3 FT	The FT is on A, B and C. Withhold B1 for each error or omission.
	Evaluate a constant, or use limits $x=1$, $t=0$ in a solution containing terms t, $a\ln x$ and $b\ln(1+2x)$, where $ab\neq 0$	M1	
	Obtain a correct expression for t in any form, e.g. $t=-\frac{1}{x}+2\ln\left(\frac{1+2x}{3x}\right)+1$	A1	
		11	